Book Reviews

Reiner Horst and Panos M. Pardalos (eds.); *Handbook of Global Optimization*, Nonconvex Optimization and Its Applications, Vol. 2, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995, ISBN 0-7923-3120-6.

During the past three decades, the field of global optimization has been growing rapidly, and there has been a concurrent and steady increase in the number of publications on all aspects of global optimization. These publications have been reporting developments in the theory, solution and application of global optimization on an ongoing basis. As the number of publications continues to grow, however, and with the development and implementation of practical global optimization algorithms, there has been an increasing need for a comprehensive, self contained reference book to provide information and references on the state of the art in global optimization theory, methods and applications. The Handbook of Global Optimization is an excellent first attempt to fulfill this need.

In the Handbook, Horst and Pardalos have assembled a total of 15 chapters written by leading scholars in global optimization from around the world. Each chapter is essentially expository in nature, yet the material is presented in a rigorous, scholarly fashion.

The first two chapters focus on two theoretical issues of importance to all global optimization problems. Hiriart-Urruty gives an overview of recent efforts to construct global optimality conditions in global optimization, while Vavasis gives a brief overview of complexity results. These two chapters immediately emphasize to the reader that, in contrast to convex programming problems, global optimization problems are generally intrinsically quite difficult to analyze and solve. In particular, it becomes quite apparent in these two chapters that, except perhaps for specially-structured problems or problems with a small number of variables, one should not expect to be able to find global optimality conditions or solution algorithms for global optimization problems that are as practical or efficient as those that have been found for convex programming problems.

Four of the chapters deal with general-purpose approaches for global optimization. In the first of these chapters, Diener gives a review of the use of trajectory methods in global optimization. These methods seek to construct paths in the domain space that pass through some or all of the global optimal solutions. Next, Forster describes the literature on the closely-related homotopy methods. These methods are used to solve complicated systems of nonlinear equations and can thus be used indirectly to solve many types of global optimization problems. Forster's chapter covers homotopy methods that are based upon either piecewise linear approximation and pivoting steps or upon differential topology and path following. In the following chapter, Ratschek and Rokne give an overview of how interval arithmetic tools and methods have been used to solve global optimization problems. Their discussion focuses on interval methods for unconstrained and constrained problems that use branch and bound searches with bisection strategies. The last of these four chapters, by Boender and Romeijn, covers stochastic methods. These methods iteratively evaluate the objective function value of a global optimization problem at random points in the feasible domain. Under mild assumptions, the probability of generating a point close to the global optimum in these methods approaches one. The chapter covers two-phase, random search and random function approaches to stochastic methods.

Three chapters are devoted to describing the importance and solution of certain fundamental classes of global optimization problems with special structure. First, Benson presents the essential elements of the theory, applications and solution algorithms of concave minimization. The problems in this class involve minimizing concave functions over closed convex sets. Benson's strategy for describing the algorithms for concave minimization is to use a classification scheme based upon the notion that each algorithm involves primarily enumeration, successive approximation or successive partitioning. Next, Tuy provides an overview of d.c. optimization problems. These are global optimization problems that can be described in terms of functions and sets that are expressible as the differences of convex functions and of convex sets, respectively. Tuy's chapter describes the theory of d.c. optimization as well as typical examples and various types of algorithms for globally solving problems in this class. The last chapter in this trilogy, by Hansen and Jaumard, summarizes the subject of Lipschitz optimization. Lipschitz optimization problems are global optimization problems with or without constraints for which the slopes of the objective functions and any functions used to help represent constraints are all bounded. In this chapter, Hansen and Jaumard describe the popular algorithms and heuristics for solving univariate and multivariate unconstrained and constrained Lipschitz optimization problems. Experimental computational comparisons are also included.

The remaining chapters focus on six important special cases of global optimization problems. Floudas and Visweswaran review recent developments in nonconvex quadratic programming, including those concerning optimality conditions, complexity, deterministic solution techniques and practical applications. Following this, Pang presents a state-of-the-art survey of the application and solution of nonlinear complementarity problems, with special attention to the closely-related topic of variational inequalities. Next, Du outlines some recent developments concerning the global solution of minimax problems of the type that seek to minimize the maximum of a finite number of continuous concave functions over a convex set. Du includes discussions of both theoretical results and applications for these important types of minimax problems. Konno and Kuno present a survey of recent proposals for globally solving multiplicative programming problems. These are minimization problems involving products of convex functions either in the objective function or in the constraints. The chapter by Schaible gives a summary of the applications, theory and global solution algorithms for various types of fractional programming problems. Included in this chapter is a comprehensive, up-to-date bibliography on the subject. Finally, the chapter by Guisewite is devoted to describing recent results concerning the complexity, application and solution of nonconvex network flow problems, with special attention to the popular minimum concave cost network flow problem.

Taken together, these 15 chapters provide an impressive overview of the field of global optimization. Each chapter is clearly written and presents a coherent and comprehensive survey of the topic at hand. In addition, the individual authors and the editors have woven the material together so that the relationships among the topics covered become evident. Furthermore, the knowledge, insight and maturity that the authors bring to their respective topics make the book a pleasure to read. Each chapter individually will undoubtedly become a valuable standard reference in its own right. Together, these chapters constitute an extraordinary sourcebook for mathematical programming specialists and for scientists of various fields who utilize global optimization models and methods. I highly recommend this outstanding volume to anyone interested in global optimization.

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Reiner Horst, Panos M. Pardalos, and Nguyen V. Thoai, *Introduction to Global Optimization*, Nonconvex Optimization and Its Applications, Vol. 3, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995, 318 pages, ISBN 0-7923-3556-2.

The purpose of this book is to provide an introduction to the basic elements of constrained global optimization. The book describes essential theory, common applications and popular approaches for global optimization in a compact, self-contained volume consisting of six chapters.

The first chapter gives a summary of some of the fundamental notions and properties of convexity, convex programming and global optimization. Included in this chapter are discussions of both classical results, such as first- and second-order optimality conditions, and more recent results concerning issues in global optimization such as the derivation of analytic formulas for convex envelopes and the complexity of finding and checking for local and global optimal solutions.

Chapter 2 focuses on the important topic of nonconvex quadratic programming. After linear programming, perhaps the most studied problem in mathematical programming is the quadratic programming problem. First, this chapter shows the importance of nonconvex quadratic programming problems in a variety of fields, including economics, finance, statistics and operations research, and that general nonconvex quadratic programs are NP-hard. Nevertheless, the authors point out that problems with certain structures or of reasonable sizes can be globally solved by appropriate algorithms. The chapter concludes by explaining the workings of several of these algorithms, including the extreme point ranking algorithm and algorithms based upon piecewise linear underestimation and branch and bound.

Chapter 3 is devoted to studying the problem of minimizing a general (i.e. not necessarily quadratic) concave function over a polytope. A sample of the many practical problems that can be formulated as concave minimization problems is presented first. Next, some of the more popular methods for solving concave minimization problems are described, including the outer approximation method, methods that use concavity cuts, the inner approximation method, and branch and bound methods. The chapter emphasizes that the ideas used in these methods are also used in many of the algorithms that have been designed for solving other types of global optimization problems.

Chapter 4 explains some of the main properties, applications and global solution methods of d.c. programming. A d.c. programming problem is a global optimization problem that can be written in terms of d.c. functions and d.c. sets, i.e., functions and sets that are expressible as differences of convex functions and of convex sets, respectively. Algorithms for certain common classes of d.c. programming problems are described here, including an edge following algorithm and two types of branch and bound algorithms.

In Chapter 5 an introduction to Lipschitz optimization is presented. Lipschitz optimization problems are global optimization problems for which the slopes of the functions used in the objective function and in any possible constraints are all bounded. These problems arise in many applications, including in the solution of best approximation problems and of systems of nonlinear equations. After describing the Lipschitz optimization problem, the chapter gives a sample of these applications and of some of the branch and bound algorithms that can be used to solve these problems.

The final chapter discusses global optimization on networks. Particular attention is paid to the minimum concave cost flow problem, for which several applications and global solution methods are described.

Each chapter contains illustrative examples and concludes with a set of carefully-selected exercises designed to enhance the depth and breadth of the reader's understanding of the material. Furthermore, care is taken to guide the reader through the material. Proofs are given as appropriate, but to keep the presentation concise, the authors do not hesitate to give intuitive arguments or to refer the reader to other references on many occasions. The mathematical prerequisites required to read this book are relatively modest. They include knowledge of linear algebra, linear programming and elementary real analysis.

Overall, this book provides an excellent introduction to the fascinating field of global optimization. The authors have used their extensive knowledge of and perspective on the field to create a coherent text that is accessible to a large audience that includes both students of mathematical programming and scientists who utilize optimization in their work. Furthermore, teachers and students of science, engineering and business methods will also find the book to be useful as a textbook in a variety of graduate and advanced undergraduate courses. I strongly recommend this book to anyone interested in an informed yet concise introduction to constrained global optimization.

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